



IB Physics — Topic 0

Uncertainties

Study notes · 完整复习笔记

1. Measurement, Precision and Uncertainty

Precision of an instrument 仪器精度

Instrument type	Precision
Analogue scale (ruler, thermometer)	$\pm \frac{1}{2}$ of the smallest division
Digital readout (balance, caliper)	± 1 in the last digit displayed

The analogue rule is a usual guide, not a law — judge each situation and make a reasonable estimate.

Example

A ruler with 0.1 cm divisions $\rightarrow$ precision = $\pm \frac{0.1 \text{ cm}}{2} = \pm 0.05 \text{ cm}$.

Reading 3.45 cm $\rightarrow L = 3.45 \pm 0.05 \text{ cm}$

A digital caliper reading to 0.01 mm $\rightarrow$ precision = $\pm 0.01 \text{ mm}$.

模拟刻度取最小分度的一半;数字显示取最后一位的1。

Expressing a measurement 测量值的表示

Measured value = quantity $\pm$ uncertainty

For a measurement $L = 15.2 \pm 0.1 \text{ cm}$:

Type of uncertainty	Definition	Value
Absolute Uncertainty 绝对不确定度	$ \pm \text{uncertainty} $	0.1 cm
Fractional Uncertainty 相对不确定度	uncertainty / quantity	$0.1 / 15.2 = 0.0066$
Percentage Uncertainty 百分不确定度	$100 \times \text{fractional}$	0.66 %

Rules for quoting a result:

- Quote the absolute uncertainty to 1 significant figure.
- Then round the quantity to the same decimal place as its uncertainty.
- Fractional uncertainty has no units; percentage uncertainty carries a % sign.
e.g. $15.237 \pm 0.1263 \text{ m}$ is written $15.2 \pm 0.1 \text{ m}$



2. Propagating Uncertainties 不确定度的传递

These three rules cover every calculation in the course. Note which uncertainty each rule uses — absolute or percentage.

Operation	Rule	Uses
Add or subtract ($C = A \pm B$)	$\Delta c = \Delta a + \Delta b$	ABSOLUTE uncertainties
Multiply or divide ($D = A \times / \div B$)	$\frac{\Delta d}{d} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$	PERCENT / fractional
Raise to a power $C = B^n$	$\frac{\Delta c}{c} = n \times \left \frac{\Delta b}{b} \right $	PERCENT $\times$ the power

口诀:加减用绝对值相加,乘除用百分比相加,幂次把百分比乘以指数。不确定度永远相加,绝不相减。

Rule 1 — addition and subtraction

If $A = a \pm \Delta a$ and $B = b \pm \Delta b$, then $A \pm B = (a \pm b) \pm (\Delta a + \Delta b)$

Example 1 — combining two masses

$$A = m_1 + m_2, \quad m_1 = 120 \pm 5 \text{ g}, \quad m_2 = 150 \pm 5 \text{ g}$$

$$\text{Quantity: } A = m_1 + m_2 = 120 + 150 = 270 \text{ g}$$

$$\text{Absolute Uncertainty: } 5 + 5 = 10 \text{ g}$$

$$\text{Total mass} = (120 + 150) \pm (5 + 5) = 270 \pm 10 \text{ g}$$

Example 2 — subtracting two lengths

$$(55 \pm 0.5 \text{ mm}) - (35 \pm 0.5 \text{ mm}) = (55 - 35) \pm (0.5 + 0.5) = 20 \pm 1 \text{ mm}$$

即使是相减,不确定度也是相加——这是最常考的陷阱。

Rule 2 — multiplication and division ($D = A \times / \div B$)

$$\frac{\Delta d}{d} = \frac{\Delta a}{a} + \frac{\Delta b}{b} \quad (\% \text{ uncertainty of product / division} = \text{sum of \% uncertainties})$$

Example 3 — area of a rectangle

$$A = L_1 \times L_2, \quad L_1 = 10 \pm 2 \text{ cm}, \quad L_2 = 12 \pm 2 \text{ cm}$$

$$L_1 = 10 \pm 2 \text{ cm} = 10 \text{ cm} \pm \frac{2}{10}, \quad L_2 = 12 \pm 2 \text{ cm} = 12 \text{ cm} \pm \frac{2}{12}$$

$$\text{Quantity: } A = 10 \times 12 = 120 \text{ cm}^2$$

$$\text{Fractional Uncertainty in D: } \frac{\Delta a}{a} = \frac{2}{10} + \frac{2}{12} = 0.367 = 36.7\%$$

$$\text{Absolute Uncertainty: } \Delta A = 120 \times 36.7\% = 44 \text{ cm}^2$$

$$A = 120 \pm 44 \text{ cm}^2, \text{ which rounded properly is } 1.2 \times 10^2 \pm 4 \times 10^1 \text{ cm}^2 \text{ (} 120 \pm 40 \text{ cm}^2 \text{)}$$

By the rule "quote the absolute uncertainty to 1 significant figure", the final answer should be written as $120 \pm 40 \text{ cm}^2$. 按“绝对不确定度保留 1 位有效数字”的规则,最终答案应写成 $120 \pm 40 \text{ cm}^2$ 。

Example 4 — density of a cylinder

$$\rho = \frac{m}{V}, \quad m = 2.00 \text{ kg} \pm 2.5\%, \quad V = 0.00100 \text{ m}^3 \pm 5\%$$

$$\text{Quantity: } \rho = m/V = 2.00 / 0.00100 = 2.00 \times 10^3 \text{ kg m}^{-3}$$

$$\% \text{ uncertainty in } \rho = 2.5\% + 5\% = 7.5\% = 0.075$$

$$\Delta \rho = 2.00 \times 10^3 \times 0.075 = 0.15 \times 10^3 \text{ kg m}^{-3}$$



$\rho = (2.00 \pm 0.15) \times 10^3 \text{ kg m}^{-3}$, which rounded properly is $\rho = (2.00 \pm 0.2) \times 10^3 \text{ kg m}^{-3}$

By the rule "quote the absolute uncertainty to 1 significant figure", the final answer should be written as $(2.00 \pm 0.2) \times 10^3 \text{ kg m}^{-3}$.

Rule 3 — powers

If $c = b^n$ then $\Delta c/c = n \cdot |\Delta b/b|$

Example 5 — elastic potential energy

$U = \frac{1}{2} k x^2$, with $k = 500 \pm 10 \text{ N m}^{-1}$ and $x = 0.25 \pm 0.02 \text{ m}$

$$k = 500 \pm 10 \text{ N m}^{-1} = 500 \text{ N m}^{-1} \pm \frac{10}{500} \text{ and } x = 0.25 \pm 0.02 \text{ m} = 0.25 \text{ m} \pm \frac{0.02}{0.25}$$

Quantity: $U = \frac{1}{2} (500)(0.25)^2 = 0.5 \times 500 \times 0.0625 = 15.625 \text{ J}$

Fractional uncertainty: $\frac{\Delta u}{u} = \frac{\Delta k}{k} + 2 \frac{\Delta x}{x} = \frac{10}{500} + 2 \frac{0.02}{0.25} = 0.18 = 18\%$

Absolute uncertainty: $\Delta U = 15.625 \times 0.18 = 2.8 \text{ J}$

$U = 15.6 \pm 2.8 \text{ J}$, or rounded properly $16 \pm 3 \text{ J}$

$\frac{1}{2}$ is an exact number — pure constants carry no uncertainty.

混合运算 When an expression mixes operations, convert every uncertainty to a percentage first, combine them with the multiply/divide and power rules, then convert back to an absolute uncertainty at the end. 混合运算先全部换成百分比,最后再换回绝对值。