EMMA YU'S DP PHYSICS
俞 老 师 课 堂

IB Physics — Topic 0
Vectors

Study notes · 完整复习笔记

1. Vectors and Scalars

Every physical quantity in this course is either a scalar or a vector.

Type	Definition
Scalar	Completely specified by a magnitude alone
Vector	Has both a magnitude and a direction

Vectors	Scalars
Displacement	Distance
Velocity	Speed (magnitude of velocity)
Acceleration	Temperature
Force and weight	Mass
Momentum and impulse	Energy, work and power
Electric and magnetic field strength	Time, volume, density, charge

注意 distance / displacement 和 speed / velocity 的区别——这是最常考的一对。

IB Physics 遇到的物理量非标量即矢量。

严格来说物理量还有张量(应力、转动惯量)、赝矢量(角动量、磁场)等类型,但 IB 不要求。

2. Notation and Properties

Notation 表示法

- A vector is written in bold (*A*) or with an arrow above it.
- In diagrams a vector is drawn as an arrow: the arrow points in the direction, and its length represents the magnitude.
- The magnitude is written with absolute-value bars $|A|$ or simply A.
- Magnitude is never negative — it is zero only for the zero vector.

Properties of vectors 矢量的性质

- Equality: two vectors are equal if they have the same magnitude AND the same direction.
- Translation: a vector may be moved parallel to itself anywhere in a diagram without changing it. Position does not matter.
- Negative vectors: two vectors are negatives of each other if they have the same magnitude but point 180° apart.
- The zero vector: magnitude zero and no defined direction. $A + (-A) = 0$.
- Only vectors of the same kind can be added — a force and a velocity can never be combined.

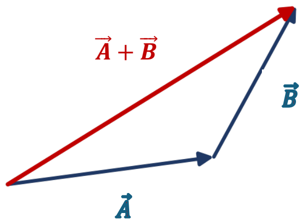
注意 矢量可以平移,所以画图时起点随便放;但方向和长度不能改。

3. Adding and Subtracting Vectors Geometrically

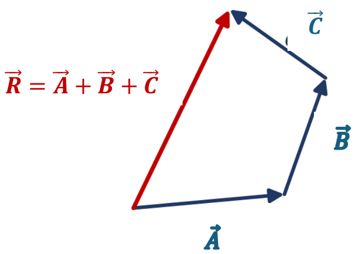
- Directions must be taken into account — you cannot simply add the magnitudes.
- The units must be the same.
- Geometric method: scale drawing. Quick and intuitive, but limited by drawing accuracy.
- Algebraic method: resolve into components. More convenient and more accurate.

Triangle and polygon methods 三角形与多边形法则

- Triangle method (tip-to-tail): move *B* so its tail sits on the tip of *A*; the resultant runs from the tail of *A* to the tip of *B*.



- Polygon method: for several vectors, draw them one after another tail-to-tip; the resultant runs from the very first tail to the very last tip.

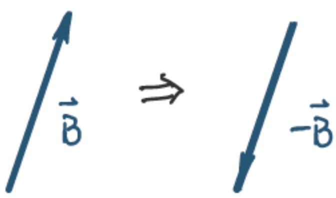


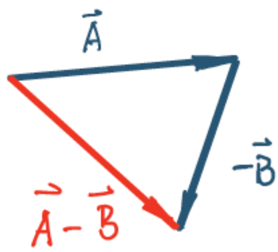
- Order does not matter: *A* + *B* = *B* + *A* (commutative), and (*A* + *B*) + *C* = *A* + (*B* + *C*) (associative).

Subtracting Vectors 矢量减法

Subtraction is a special case of addition: $A-B=A+(-B)$

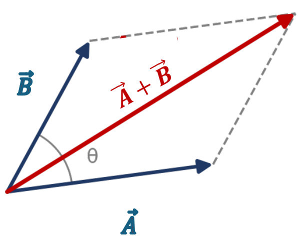
Rotate B by 180o to get $-B$, then add it to A tail to tip in the usual way.



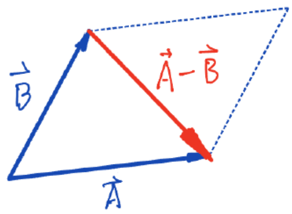


Parallelogram law 平行四边形法则

- Draw the two vectors tail-to-tail as adjacent sides of a parallelogram; the diagonal from their common tail is *A* + *B*.



- The other diagonal represents *A* − *B*.

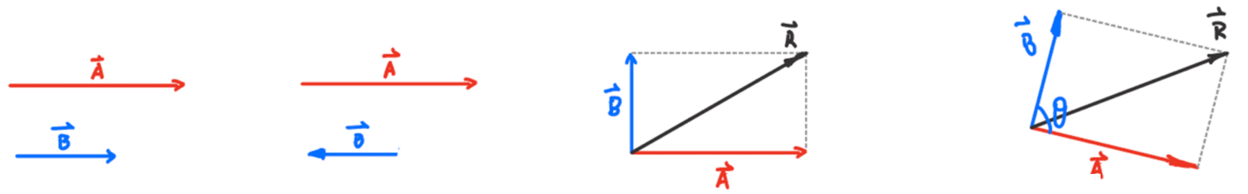


Closed polygon = zero resultant 闭合多边形

- If the vectors drawn tail-to-tip close up into a polygon, their resultant is zero.
- For forces this is the condition for translational equilibrium — three forces in equilibrium must form a closed triangle.

Special cases and the cosine rule 特殊情况与余弦定理

- Same direction (θ = 0°): *R* = *A* + *B* — the largest possible resultant.
- Opposite directions (θ = 180°): *R* = |*A* − *B*| — the smallest possible resultant.
- Perpendicular (θ = 90°): $R = \sqrt{A^2 + B^2}$, direction $\tan^{-1}(B/A)$.



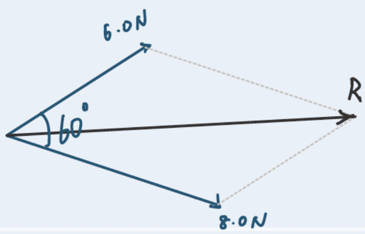
R = *A* + *B*   *R* = |*A* − *B*| $R = \sqrt{A^2 + B^2}$ $R = \sqrt{\quad {}^2 + 2AB\cos\theta}$

- Any angle: use the cosine rule.

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}\,, \qquad |A - B| \le R \le A + B$$

θ 是两矢量尾对尾时的夹角,所以公式里是 +2AB cos θ。如果用的是三角形的内角(180° − θ),符号要变成减号——先在图上标清楚角度再套公式。

Example : Two forces of 6.0 N and 8.0 N act at a point, 60° apart. Find the resultant force.



Magnitude: $R = \sqrt{6.0^2 + 8.0^2 + 2 \times 6.0 \times 8.0 \times \cos 60°} = \sqrt{(36 + 64 + 48)} = \sqrt{148} = 12.2\ N$

Direction, measured from the 8.0 N force:

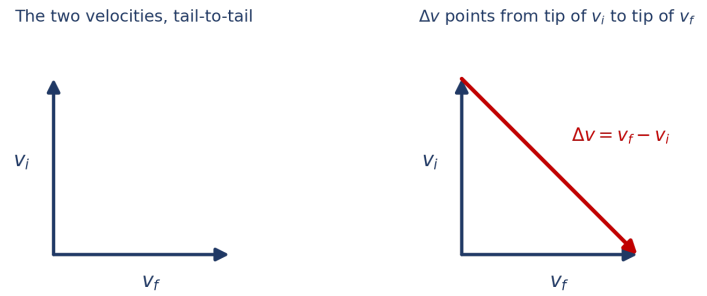
tan α = 6.0 sin 60° / (8.0 + 6.0 cos 60°) = 5.196 / 11.0 = 0.4724 → α = 25.3°

Answer: **12 N at 25° from the 8.0 N force.**

检查 合力一定落在 |8 − 6| = 2.0 N 和 8 + 6 = 14.0 N 之间,12.2 N 合理 ✓

4. Change in a vector quantity

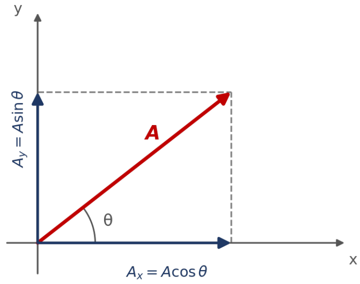
- $\Delta v = v_f - v_i$. Draw the two velocities tail-to-tail; Δv is the arrow from the tip of v_i to the tip of v_f.



5. Components of a Vector

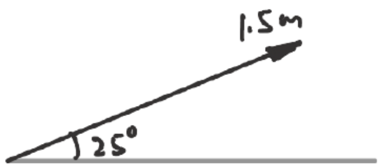
A component is the projection of a vector onto a coordinate axis.
Rectangular components are the projections onto the *x*- and *y*-axes.
θ is measured anticlockwise from the positive *x*-axis.

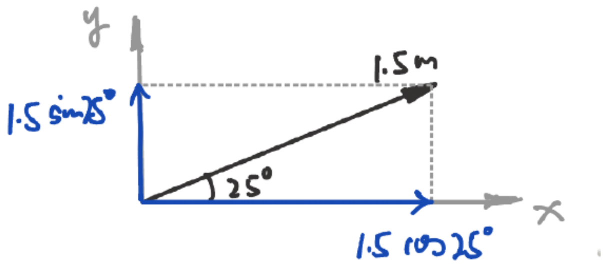
$$\begin{cases} A_x = A\cos\theta \\ A_y = A\sin\theta \end{cases} \qquad (\theta \text{ measured from the +x axis})$$



If it is measured from the *y*-axis instead, sine and cosine swap over — always mark the angle on your diagram first.

A vector can therefore be described in two equivalent ways:





- By magnitude and direction — e.g. a displacement of 1.5 m at an angle of 25°.
- By components — e.g. 1.36 m in the +*x* direction and 0.634 m in the +*y* direction.
  Check: 1.5 cos 25° = 1.36 m, 1.5 sin 25° = 0.634 m ✓

6. Adding Vectors by Components

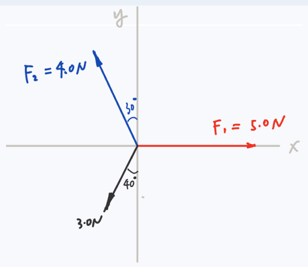
This is the method to use whenever the vectors are not at right angles.

- Resolve every vector into its *x*- and *y*-components (watch the signs — left and down are negative).
- Add the components separately.
- Recombine into a magnitude and a direction.
  注意 A calculator's $\tan^{-1}$ only returns angles between −90° and +90°, so check the signs of R_x and R_y to place the resultant in the correct quadrant. 计算器给的反正切只在 ±90° 之间,要根据分量正负判断象限

$$\boldsymbol{R}_x = \boldsymbol{A}_x + \boldsymbol{B}_x \qquad \boldsymbol{R}_y = \boldsymbol{A}_y + \boldsymbol{B}_y$$

$$\begin{cases} \boldsymbol{R} = \sqrt{\boldsymbol{R}_x^{\,2} + \boldsymbol{R}_y^{\,2}} \\ \theta = \tan^{-1}\left(\dfrac{R_y}{R_x}\right) \end{cases}$$

Example: Three forces act at a point: 5.0 N at 0°, 4.0 N at 120° and 3.0 N at 230° (all measured from the +x axis). Find the resultant.



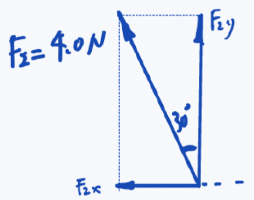
Components:

5.0 N at 0°

$F_{1x} = 5.0\,N$

$F_{1y} = 0\,N$

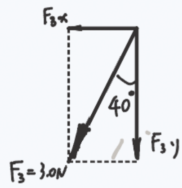
4.0 N at 120°



$F_{2x} = -4.0 \sin 30° = -2.0\,N$

$F_{2y} = 4.0 \cos 30° = 3.464\,N$

3.0 N at 230°



$F_{3x} = -3.0 \sin 40° = -1.928\,N$

$F_{3y} = -3.0 \cos 40° = -2.298\,N$

$R_x = 5.000 - 2.000 - 1.928 = 1.072\ N$

$R_y = 0.000 + 3.464 - 2.298 = 1.166\ N$

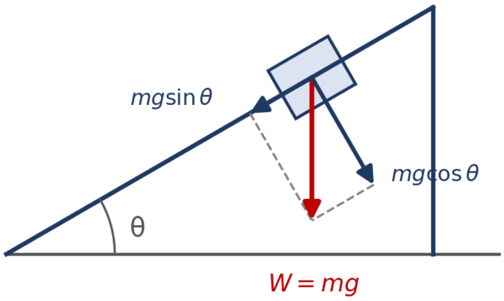
$R = \sqrt{1.072^2 + 1.166^2} = 1.58\ N$

$\theta = tan^{-1}(1.166\ /\ 1.072) = 47.4°$ — both components positive, so the first quadrant ✓

Answer: 1.6 N at 47° from the +x axis.

7. Resolving on an Inclined Plane

On a slope it is much easier to use axes parallel and perpendicular to the surface, because the motion happens along the surface.



- θ is the angle of the incline. The weight makes the same angle θ with the normal to the surface, which is why the perpendicular component takes the cosine.
- The weight needs to be decomposed .

$$\begin{cases} \text{along the slope} : mgsin\theta \\ \text{perpendicular to the slope} : mgcos\theta \end{cases}$$